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MOTION OF ELECTRON GAS IN CONDUCTING SOLIDS

By H. Demiray and A. C. Eringen

Technical Report No. 29

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H. Demiray and A. C. Eringen

Department of Aerospace and Mechanical Sciences

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MOTION OF ELECTRON GAS IN CONDUCTING SOLIDS*

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ABSTRACT

By use of the continuum theory of mixtures, a theory is presented for elastic conductors. The ionic lattice and electronic gas as separate continua constitute the members of the mixture. Balance laws and jump conditions are given, and a set of properly invariant constitutive equations are obtained and linearized. The field equations are solved for a problem of electrostatic probe, namely, a charged spherical cavity in a solid plasma.

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1. INTRODUCTION

In recent years, a revival of interest has reopened research in the field of electromagnetic elastic solids. Besides growing technological importance, certain fundamental questions hitherto unresolved gave impetus to these studies. Among them are the nature of electromagnetic force, energy, and the question of invariance of the constitutive functions. In continuum theories, Euclidean invariance and the principle of objectivity are taken as fundamental, while in electromagnetic theory, the Lorentz invariance is basic. For deformable bodies subject to electromagnetic fields, a unification and reconciliation of these two points of view was necessary (cf. Grot and Eringen [1], Jordan and Eringen [2], and Walker [3]). In these works, a continuum was assumed to possess no electron inertia, so that the electromagnetic force on the current was transmitted directly to the solid continuum. In reality, in fact, the force acting on the current is transmitted to a solid continuum through momentum transferred by the electrons.

In a recent work [4], we presented a continuum theory of charged mixtures capable of diffusion, ionization, and recombination. In this work, the effect of the diamagnetic property of electrons and ions was also included. Here we consider a mixture composed of polarizable and magnetizable conductors and an electron gas continuum that possesses inertia.

Theory developed here is believed to have potential application in the discussion of electrical charges in solids in particular in lightening damage, although the latter problem requires the inclusion of thermal effects, in the discussion of near fields.

2. BALANCE EQUATIONS

The balance equations of a mixture composed of an elastic conductor and an electron gas continuum may be obtained as a special case of equations (2.1) to (4.6) of reference [4], with the constraints that

$$C_{(\alpha)} = 0 \quad , \quad \underset{\sim}{\Lambda}_{(\alpha)} = 0 \quad , \quad \alpha = e,s$$

(2.1)

$$\underset{\sim}{P}_{(\alpha)} = 0 \quad , \quad \underset{\sim}{M}_{(\alpha)} = 0 \quad , \quad \alpha = e$$

where $C_{(\alpha)}$ and $\underset{\sim}{\Lambda}_{(\alpha)}$ are respectively the mass production (or destruction) and the rate of polarization transfer Λ, $\underset{\sim}{P}_{(\alpha)}$ is the partial polarization, and $\underset{\sim}{M}_{(\alpha)}$ is the partial magnetization of species α which takes the values s and e associating the fields, respectively, with the ionic lattice and electronic gas continua. The balance laws and jump conditions are:

(i) <u>Conservation of Mass</u>:

(2.2) $$\frac{\partial \rho_{(\alpha)}}{\partial t} + \mathrm{div}(\rho_{(\alpha)} \underset{\sim}{v}_{(\alpha)}) = 0 \qquad \text{in } V_{(\alpha)} - \sigma$$

(2.3) $$\underset{\sim}{[} \rho_{(\alpha)} (\underset{\sim}{v}_{(\alpha)} - \underset{\sim}{u}) \underset{\sim}{]} \cdot n = 0 \qquad \text{on } \sigma$$

where $\rho_{(\alpha)}$, $\underset{\sim}{v}_{(\alpha)}$ are respectively the density and the velocity of the α th species, and $\underset{\sim}{u}$ is the velocity of a moving discontinuity surface σ with exterior normal $\underset{\sim}{n}$.

(ii) <u>Balance of Momentum</u>:

(2.4) $$t^{k\ell}_{(\alpha);k} + g^{\ell}_{(\alpha)} + \rho_{(\alpha)}(f^{\ell}_{(\alpha)} - \dot{v}^{\ell}_{(\alpha)}) = R^{\ell}_{(\alpha)} \quad \text{in } V_{(\alpha)} - \sigma$$

(2.5) $$\underset{\sim}{[} \rho_{(\alpha)}(v^{k}_{(\alpha)} - u^{k}) v^{\ell}_{(\alpha)} - t^{k\ell}_{(\alpha)} \underset{\sim}{]} n_k = \hat{R}^{\ell}_{(\alpha)} \qquad \text{on } \sigma$$

with $R^k_{(\alpha)}$ and $\hat{R}^k_{(\alpha)}$ subject to

(2.6) $$\sum_\alpha R^k_{(\alpha)} = 0 \quad , \quad \sum_\alpha \hat{R}^k_{(\alpha)} = 0$$

where $\underset{\sim}{t}_{(\alpha)}$, $\underset{\sim}{f}_{(\alpha)}$, $\underset{\sim}{R}_{(\alpha)}$, $\underset{\sim}{\hat{R}}_{(\alpha)}$ and $\underset{\sim}{g}_{(\alpha)}$ are, respectively, the stress tensor, mechanical body force, the linear momentum transfer in volume $V_{(\alpha)} - \sigma$ and on the surface σ, and the body force due to electromagnetic origin defined by

(2.7)
$$g_i^{(s)} \equiv q_{(s)}E_i^{(s)} + E^{(s)}_{i;m}P^m_{(s)} + \varepsilon_{ijk}\overset{*}{P}{}^j_{(s)}B^k + M_k^{(s)}B^k_{;i}$$
$$g_i^{(e)} \equiv q_{(e)}E_i^{(e)}$$

Here $q_{(\alpha)}$, $\underset{\sim}{E}_{(\alpha)}$, $\underset{\sim}{B}$, $\underset{\sim}{M}_{(\alpha)}$, $\underset{\sim}{P}_{(\alpha)}$ and $\underset{\sim}{\overset{*}{P}}_{(\alpha)}$ are respectively the charge density, effective electric field, magnetic field, magnetization, polarization, and the convective time rate of polarization defined by

(2.8)
$$\underset{\sim}{E}_{(\alpha)} \equiv \underset{\sim}{E} + \frac{\underset{\sim}{v}_{(\alpha)}}{c} \times \underset{\sim}{B}$$
$$\overset{*}{P}{}^i_{(\alpha)} \equiv \hat{P}^i_{(\alpha)} + P^i_{(\alpha)}v^{(\alpha)r}{}_{;r} - P^j_{(\alpha)}v^i_{(\alpha);j}$$

Throughout this work, a semi-colon is used to indicate the covariant partial differentiation and a superposed prime the material derivative of tensor fields following the motion of the α th species.

(iii) <u>Balance of Moment of Momentum</u>:

(2.9) $$t^{k\ell}_{(\alpha)} - t^{\ell k}_{(\alpha)} + \varepsilon^{k\ell m}(G_m^{(\alpha)} + T_m^{(\alpha)}) = 0 \quad \text{in } V_{(\alpha)} - \sigma$$

(2.10) $$\hat{T}^k_{(\alpha)} = 0 \qquad \text{on } \sigma$$

with $T^k_{(\alpha)}$ and $\hat{T}^k_{(\alpha)}$ subject to

$$\sum_\alpha T^k_{(\alpha)} = 0 \quad , \sum_\alpha \hat{T}^k_{(\alpha)} = 0 \tag{2.11}$$

where $T_{(\alpha)}$, $\hat{T}_{(\alpha)}$, and $G^k_{(\alpha)}$ are respectively the rates of angular momentum transfer in volume $V_{(\alpha)}-\sigma$ and on the surface σ , and the electromagnetic couple defined by

$$\underset{\sim}{G}_{(s)} \equiv \underset{\sim}{P}_{(s)} \times \underset{\sim}{E}_{(s)} + \underset{\sim}{M}_{(s)} \times \underset{\sim}{B} \quad , \quad \underset{\sim}{G}_{(e)} \equiv 0 \tag{2.12}$$

(iv) Conservation of Energy:

$$\rho_{(s)}\dot{\varepsilon}_{(s)} = t^{k\ell}_{(s)} v^{(s)}_{\ell;k} + \rho_{(s)} E^{(s)}_i \left(\frac{P^i_{(s)}}{\rho_{(s)}}\right)^{\boldsymbol{\cdot}} - M^i_{(s)} \dot{B}_i + \frac{1}{2\theta_{(s)}} R^\ell_{(e)} u^{(es)}_\ell + q^k_{(s);k} + \rho_{(s)} h_{(s)} + e_{(s)} \tag{2.13}$$

$$\rho_{(e)}\dot{\varepsilon}_{(e)} = t^{k\ell}_{(e)} v^{(e)}_{\ell;k} + \frac{1}{2\theta_{(e)}} R^\ell_{(e)} u^{(es)}_\ell + q^k_{(e);k} + \rho_{(e)} h_{(e)} + e_{(e)} \tag{2.14}$$

$$[\rho_{(\alpha)}(\varepsilon_{(\alpha)} + \tfrac{1}{2} v^2_{(\alpha)})(v^k_{(\alpha)} - u^k) - t^{k\ell}_{(\alpha)} v^{(\alpha)}_\ell - q^k_{(\alpha)}] n_k = \hat{e}_{(\alpha)}, \text{ on } \sigma \tag{2.15}$$

where $e_{(\alpha)}$ and $\hat{e}_{(\alpha)}$ are subject to

$$\sum_\alpha e_{(\alpha)} = 0 \quad , \sum_\alpha \hat{e}_{(\alpha)} = 0 \tag{2.16}$$

Here $\varepsilon_{(\alpha)}$, $h_{(\alpha)}$, $\underset{\sim}{q}_{(\alpha)}$, $e_{(\alpha)}$ and $\hat{e}_{(\alpha)}$ are respectively the internal energy density, volume heat supply, surface heat flux, and the values of the rate of energy transfer in volume $V_{(\alpha)}-\sigma$ and on the surface σ of the α th component.

(v) Principle of Entropy:

$$\rho_{(\alpha)}\eta'_{(\alpha)} - \left(\frac{q^k_{(\alpha)}}{\theta_{(\alpha)}}\right)_{;k} - \frac{\rho_{(\alpha)}h_{(\alpha)}}{\theta_{(\alpha)}} + n_{(\alpha)} \geq 0, \text{ in } V_{(\alpha)} - \sigma \tag{2.17}$$

$$[\rho_{(\alpha)}\eta_{(\alpha)}(v^k_{(\alpha)} - u^k) - \frac{q^k_{(\alpha)}}{\theta_{(\alpha)}}]n^k + \hat{n}_{(\alpha)} \geq 0 \quad \text{on } \sigma \tag{2.18}$$

where $n_{(\alpha)}$ and $\hat{n}_{(\alpha)}$ are subject to

$$\sum_\alpha n_{(\alpha)} \geq 0 \quad , \quad \sum_\alpha \hat{n}_{(\alpha)} = 0 \tag{2.19}$$

Here $\eta_{(\alpha)}$, $n_{(\alpha)}$ and $\hat{n}_{(\alpha)}$ are respectively the entropy volume density and the rate of entropy transfer in volume $V_{(\alpha)} - \sigma$ and on the surface σ of the α th component.

For convenience we introduce the following Legendre transformation

$$\begin{aligned} \Psi_{(s)} &\equiv \varepsilon_{(s)} - \theta^{(s)}\eta_{(s)} + \frac{\underset{\sim}{M}_{(s)}\cdot\underset{\sim}{B}}{\rho_{(s)}} \\ \Psi_{(e)} &\equiv \varepsilon_{(e)} - \theta^{(e)}\eta_{(e)} \end{aligned} \tag{2.20}$$

Carrying (2.20) into (2.17) and eliminating the $\rho_{(\alpha)}h_{(\alpha)}$ from (2.17), by use of (2.13) and (2.14), we obtain a new form for the energy inequality

$$\begin{aligned} -\frac{\rho_{(s)}}{\theta_{(s)}}(\eta_{(s)}\theta'_{(s)} + \Psi'_{(s)}) + \frac{1}{\theta_{(s)}}\bar{t}^{k\ell}_{(s)}v^{(s)}_{\ell;k} + \frac{1}{\theta_{(s)}}E^i_{(s)}\dot{P}^{(s)}_i + \frac{1}{\theta_{(s)}}B^i\dot{M}^{(s)}_i \\ + \frac{1}{2\theta_{(s)}}R^\ell_{(e)}u^{(es)}_\ell + \frac{q^k_{(s)}\theta^{(s)}_{;k}}{\theta^2_{(s)}} + \frac{e_{(s)}}{\theta_{(s)}} + n_{(s)} \geq 0 \end{aligned} \tag{2.21}$$

$$\begin{aligned} -\frac{\rho_{(e)}}{\theta_{(e)}}(\eta_{(e)}\theta'_{(e)} + \Psi'_{(e)}) + \frac{1}{\theta_{(e)}}t^{k\ell}_{(e)}v^{(e)}_{\ell;k} + \frac{1}{2\theta_{(e)}}R^\ell_{(e)}u^{(es)}_\ell \\ + \frac{q^k_{(e)}\theta^{(e)}_{;k}}{\theta^2_{(e)}} + \frac{e_{(e)}}{\theta_{(e)}} + n_{(e)} \geq 0 \end{aligned} \tag{2.22}$$

Here $\bar{t}^{k\ell}_{(s)}$ is defined by

$$\bar{t}^{k\ell}_{(s)} \equiv t^{k\ell}_{(s)} + (\underset{\sim}{P}_{(s)} \cdot \underset{\sim}{E}^{(s)} + \underset{\sim}{M}_{(s)} \cdot \underset{\sim}{B}) g^{k\ell} \tag{2.23}$$

where $g^{k\ell}$ is the metric tensor of the spatial frame of reference $\underset{\sim}{x}$. These inequalities must be valid for all <u>independent processes</u>.

(vi) <u>Electromagnetic Field Equations</u>:

$$\nabla \times \underset{\sim}{E} + \frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} = 0 \quad , \quad [\underset{\sim}{E}] \times \underset{\sim}{n} + \frac{1}{c}[\underset{\sim}{B}]\underset{\sim}{u} \cdot \underset{\sim}{n} = 0 \tag{2.24}$$

$$\nabla \times \underset{\sim}{H} = \frac{1}{c}\frac{\partial \underset{\sim}{D}}{\partial t} + \frac{4\pi}{c}\underset{\sim}{J}, \quad [\underset{\sim}{H}] \times \underset{\sim}{n} = \frac{1}{c}[\underset{\sim}{D}]\underset{\sim}{u} \cdot \underset{\sim}{n} + \frac{4\pi}{c}\hat{\underset{\sim}{J}} \tag{2.25}$$

$$\nabla \cdot \underset{\sim}{D} = 4\pi q \quad , \quad [\underset{\sim}{D}] \cdot \underset{\sim}{n} = 4\pi\hat{q} \tag{2.26}$$

$$\nabla \cdot \underset{\sim}{B} = 0 \quad , \quad [\underset{\sim}{B}] \cdot \underset{\sim}{n} = 0 \tag{2.27}$$

where $\underset{\sim}{E}$, $\underset{\sim}{D}$, $\underset{\sim}{H}$, $\underset{\sim}{B}$, $\underset{\sim}{J}$, q, $\hat{\underset{\sim}{J}}$ and $\hat{q}$ are respectively the electric field, electric displacement, magnetic field, magnetic induction, and the values of the current and charge in volume $V-\sigma$ and on the surface of discontinuity σ.

These quantities are related to each other by

$$\underset{\sim}{D} \equiv \underset{\sim}{E} + 4\pi\underset{\sim}{P}_{(s)} \quad , \quad \underset{\sim}{H} \equiv \underset{\sim}{B} - 4\pi(\underset{\sim}{M}_{(s)} + \frac{\underset{\sim}{v}_{(s)}}{c} \times \underset{\sim}{P}_{(s)})$$

$$\underset{\sim}{J} \equiv q_{(s)}\underset{\sim}{v}_{(s)} + q_{(e)}\underset{\sim}{v}_{(e)} \tag{2.28}$$

$$q \equiv q_{(s)} + q_{(e)}$$

3. CONSTITUTIVE EQUATIONS

We select velocity, magnetization, and polarization of a solid as well as the density of electron gas and the first gradient of deformation of an elastic solid continuum as independent constitutive variables, i.e.,

$$v^k_{(\beta)},\ P^k_{(s)},\ M^k_{(s)},\ \frac{\partial x^k}{\partial X^K_{(s)}} \equiv x^k_{,K},\ \rho_{(e)},\ \theta_{(\beta)} \tag{3.1}$$

The dependent variables are:

$$t^{k\ell}_{(\alpha)},\ \Psi_{(\alpha)},\ \eta_{(\alpha)},\ q^k_{(\alpha)},\ R^k_{(\alpha)},\ E^k_{(s)},\ B^k,\ e_{(\alpha)},\ n_{(\alpha)} \tag{3.2}$$

The constitutive equations that obey the principle of equipresence have the following functional forms,

$$t^{k\ell}_{(\alpha)} = t^{k\ell}_{(\alpha)}(\rho_{(e)},\ \theta_{(\beta)},\ u^k_{(es)},\ P^k_{(s)},\ M^k_{(s)},\ x^k_{,K}) \tag{3.3}$$

similar forms being valid for other constitutive variables.

The local Clausius-Duhem inequality imposes the following restrictions on the free energies

$$\begin{aligned} \Psi_{(s)} &= \Psi_{(s)}(\theta_{(s)},\ P_{(s)},\ M_{(s)},\ x^k_{,K}) \\ \Psi_{(e)} &= \Psi_{(e)}(\theta_{(e)},\ \rho_{(e)}) \end{aligned} \tag{3.4}$$

Substitution of (3.4) into (2.21) and (2.22) yields

$$- \frac{\rho_{(s)}}{\theta_{(s)}} \left(\eta_{(s)} + \frac{\partial\Psi_{(s)}}{\partial\theta_{(s)}}\right)\dot{\theta}_{(s)} + \frac{1}{\theta_{(s)}} \left(t^{k\ell}_{(s)} - \rho_{(s)} \frac{\partial\Psi_{(s)}}{\partial x_{\ell,K}} x^k_{,K}\right) v^{(s)}_{\ell;k}$$

(3.5)
$$+ \frac{1}{\theta_{(s)}} \left(E^i_{(s)} - \rho_{(s)} \frac{\partial\Psi_{(s)}}{\partial P^{(s)}_i}\right)\dot{P}^{(s)}_i + \frac{1}{\theta_{(s)}} \left(B^i - \rho_{(s)} \frac{\partial\Psi_{(s)}}{\partial M^{(s)}_i}\right)\dot{M}^{(s)}_i$$

$$+ \frac{1}{2\theta_{(s)}} R^{\ell}_{(e)} u^{(es)}_{\ell} + \frac{q^k_{(s)}\theta^{(s)}_{;k}}{\theta^2_{(s)}} + \frac{e_{(s)}}{\theta_{(s)}} : \eta_{(s)} \geq 0$$

A similar form may be given for the electron continuum.

Further simplification of (3.5) can be accomplished if we note that $v^{(\alpha)}_{\ell;k}$, $\dot{P}^{(s)}_i$, $\dot{M}^{(s)}_i$ and $\theta^{(\alpha)}_{;k}$ may be varied arbitrarily. The necessary and sufficient conditions for the inequality (3.5) to be valid for all arbitrary variations of these quantities is that the coefficients of these quantities must vanish, i.e.,

(3.6)
$$\eta_{(\alpha)} = - \frac{\partial\Psi_{(\alpha)}}{\partial\theta_{(\alpha)}} \quad , \quad q^k_{(\alpha)} = 0 \quad , \quad \alpha = e,s$$

(3.7)
$$t^{k\ell}_{(e)} = -\rho^2_{(e)} \frac{\partial\Psi_{(e)}}{\partial\rho_{(e)}} g^{k\ell}$$

$$t^{k\ell}_{(s)} = \rho_{(s)} \frac{\partial\Psi_{(s)}}{\partial x_{\ell,K}} x^k_{,K} - (\underset{\sim}{P}_{(s)} \cdot \underset{\sim}{E}^{(s)} + \underset{\sim}{M}_{(s)} \cdot \underset{\sim}{B}) g^{k\ell}$$

(3.8)
$$E^k_{(s)} = \rho_{(s)} \frac{\partial\Psi_{(s)}}{\partial P^{(s)}_k} \quad , \quad B^k = \rho_{(s)} \frac{\partial\Psi_{(s)}}{\partial M^{(s)}_k}$$

subject to

(3.9)
$$\varepsilon_{k\ell m}\left(\frac{\partial\Psi_{(s)}}{\partial P^{(s)}_k} P^{\ell}_{(s)} + \frac{\partial\Psi_{(s)}}{\partial M^{(s)}_k} M^{\ell}_{(s)} + \frac{\partial\Psi_{(s)}}{\partial x_{k,K}} x^{\ell}_{,K}\right) = 0$$

At this point, a remark is in order: Since we are dealing with the dynamics of deformable electromechanical materials, then a question

arises as to whether the free energy density is invariant under transformations of the spatial frame of reference or not. Clearly, it is not unless one replaces $P^k_{(s)}$ and $M^k_{(s)}$ by $\mathcal{P}^k_{(s)}$ and $\mathcal{M}^k_{(s)}$, where $\mathcal{P}^k_{(s)}$ and $\mathcal{M}^k_{(s)}$ are respectively the values of polarization and magnetization measured on the coordinate frame moving with material body. However, the energy equation is written in terms of $P^k_{(s)}$ and $M^k_{(s)}$, therefore, such a replacement is not suitable because of the restrictions imposed by the second law of thermodynamics. Another way of handling this difficulty is to introduce a fictitious rate of electromagnetic momentum so as to obtain an energy equation written in terms of $\mathcal{P}^k_{(s)}$ and $\mathcal{M}^k_{(s)}$. The electromagnetic momentum so introduced is not unique and requires modification of Cauchy's law, thus opening new questions. As a result, the question of invariance requirements for deformable bodies subject to electromagnetic fields is still an open question and must be studied separately. In this regard, we refer the readers to works by Dixon and Eringen [5] and Toupin [6] who eventually used similar formulations that we employ here. The present formulation is certainly applicable for cases where the component of the magnetic flux perpendicular to the particle velocity is very small as compared to the component parallel to the particle velocity.

Carrying (3.6) to (3.9) into (3.5), the inequalities (3.5) are simplified to

$$\frac{1}{2\theta_{(e)}} R^{\ell}_{(e)} u^{(es)}_{\ell} + \frac{e_{(e)}}{\theta_{(e)}} + n_{(e)} \geq 0 \tag{3.10}$$

$$\frac{1}{2\theta_{(s)}} R^{\ell}_{(e)} u^{(es)}_{\ell} + \frac{e_{(s)}}{\theta_{(s)}} + n_{(s)} \geq 0 \tag{3.11}$$

The principle of objectivity requires that $\Psi_{(s)}$ must have the following form

$$\Psi_{(s)} = \Psi_{(s)}\left(\theta^{(s)}, E_{KL}, P_K, M_K\right) \tag{3.12}$$

where E_{KL}, P_K, and M_K are defined by

$$2E_{KL} = g_{k\ell}x^k_{,K}x^\ell_{,L} - G_{KL}$$
$$P_K = g_{k\ell}x^K_{,K}P^\ell_{(s)}\ , \quad M_K = g_{k\ell}x^K_{,K}M^\ell_{(s)} \tag{3.13}$$

Here E_{KL} is the Lagrangian strain tensor defined on the material coordinate system X^K whose metric tensor is G_{KL}. The constitutive equations for the stress tensor, electric field, and magnetic flux now take the forms

$$t^{k\ell}_{(s)} = \rho_{(s)}\frac{\partial\Psi_{(s)}}{\partial E_{KL}}x^k_{,K}x^\ell_{,L} + E^k_{(s)}P^\ell_{(s)} + B^kM^\ell_{(s)}$$
$$- (\underset{\sim}{P}_{(s)}\cdot\underset{\sim}{E}_{(s)} + \underset{\sim}{M}_{(s)}\cdot\underset{\sim}{B})g^{k\ell} \tag{3.14}$$

$$E^k_{(s)} = \rho_{(s)}\frac{\partial\Psi_{(s)}}{\partial P_K}x^k_{,K}\quad , \qquad B^k = \rho_{(s)}\frac{\partial\Psi_{(s)}}{\partial M_K}x^k_{,K} \tag{3.15}$$

Here, it is seen from (3.14) that the condition (2.9) is satisfied if

$$T^k_{(\alpha)} = 0 \quad , \qquad \alpha = e,s \tag{3.16}$$

From this general formulation, various order constitutive equations may be obtained. In particular for isotropic materials, the stress tensor, electric and magnetic fields can be expressed in terms of certain invariants

of independent state variables. Such a case has been given by Grot and Eringen [1] for a single medium, therefore we do not repeat them here. In what follows we will be interested in the linear constitutive equations.

4. LINEAR CONSTITUTIVE EQUATIONS

For the linear constitutive theory, the difference between material and spatial coordinates may be disregarded. In this case, the constitutive equations presented in (3.14) to (3.15) reduce to

$$(4.1)\quad t^{k\ell}_{(s)} = \rho_{(s)}\frac{\partial\Psi_{(s)}}{\partial e_{k\ell}} + E^{k}_{(s)}P^{\ell}_{(s)} + B^{k}M^{\ell}_{(s)} - (\underset{\sim}{P}_{(s)}\cdot\underset{\sim}{E}_{(s)}+\underset{\sim}{M}_{(s)}\cdot\underset{\sim}{B})g^{k\ell}$$

$$(4.2)\quad E^{k}_{(s)} = \rho_{(s)}\frac{\partial\Psi_{(s)}}{\partial P^{(s)}_{k}} \quad , \quad B^{k} = \rho_{(s)}\frac{\partial\Psi_{(s)}}{\partial M^{(s)}_{k}}$$

Here, the infinitesimal strain tensor $e^{k\ell}$ is defined by

$$(4.3)\quad e_{k\ell} \equiv \tfrac{1}{2}\,(u_{k;\ell}+u_{\ell;k})$$

where $\underset{\sim}{u}$ is the displacement vector.

For linearly isotropic materials, we express the free energy of solid continuum as

$$(4.4)\quad \Psi_{(s)} = \frac{1}{2\rho^{0}_{(s)}}\;(e^{r}_{r})^{2} + \frac{\mu}{\rho^{0}_{(s)}}\,e^{k\ell}e_{k\ell} + \frac{\chi_1}{2\rho_{(s)}}\,P^{2}_{(s)} + \frac{\chi_2}{2\rho_{(s)}}\,M^{2}_{(s)}$$

where λ , μ , χ_1 , χ_2 are the material constants, and $\rho^{0}_{(s)}$ and $\rho_{(s)}$ are, respectively the mass density in the undeformed and deformed bodies. The

$$(4.5)\quad \rho_{(s)} = \rho^{0}_{(s)}\;(1-e^{r}_{r})$$

Inserting (4.4) into (4.1) and (4.2), and using (4.5), the linear constitutive equations are obtained to be

$$(4.6)\quad E^{k}_{(s)} = \chi_1\,P^{k}_{(s)} \quad , \quad B^{k} = \chi_2\,M^{k}_{(s)}$$

$$t^{k\ell}_{(s)} = \lambda e^{r}_{r}\, g^{k\ell} + 2\mu\, e^{k\ell} + E^{k}_{(s)}\, P^{\ell}_{(s)} + B^{k}\, M^{\ell}_{(s)}$$

(4.7)

$$- \frac{1}{2}\,(\underset{\sim}{P}_{(s)}\cdot\underset{\sim}{E}_{(s)}+\underset{\sim}{M}_{(s)}\cdot\underset{\sim}{B})g^{k\ell}$$

Here we have neglected the powers of infinitesimal strain tensor higher than the first.

In equation (4.7) it is interesting to note that the total stress tensor $t^{k\ell}_{(s)}$ of solid continuum consists of two parts: The first part is the classical Hook's law, and the second part is none other than Maxwell stress tensor.

In the linear theory, the forms of the rate of linear momentum transfer, energy and entropy transfers are (cf. [4])

(4.8)
$$R^{k}_{(e)} = \nu\, u^{k}_{(es)} - \tau\, P^{k}_{(s)}$$

$$e_{(e)} = \beta_o\,(\theta_{(s)}-\theta_{(e)}) + \beta_1\, u^2_{(es)} + \beta_2\, e^r_r + \beta_3\,(e^r_r)^2$$

(4.9)

$$+ \beta_4 e^{k\ell} e_{k\ell} + \beta_5 M^2_{(s)} + \beta_6 P^2_{(s)} + \beta_7 \underset{\sim}{u}_{(es)}\cdot\underset{\sim}{P}_{(s)}$$

$$\eta_{(e)} = -\frac{\beta_o(\theta_{(s)}-\theta_{(e)})}{\theta} + \gamma_1 u^2_{(es)} + \gamma_2 e^r_r + \gamma_3 (e^r_r)^2$$

(4.10)

$$+ \gamma_4 e^{k\ell} e_{k\ell} + \gamma_5 M^2_{(s)} + \gamma_6 P^2_{(s)} + \gamma_7 \underset{\sim}{u}_{(es)}\cdot\underset{\sim}{P}_{(s)}$$

Material moduli ν , τ , β_o to β_7 and γ_1 to γ_7 appearing in (4.8) to (4.10) are functions of density of electron continuum.

The local Clausius-Duhem inequality imposes the following restrictions on these coefficients

$$\beta_o \geq 0 \; ; \; \frac{1}{2}\nu \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_1 \mp \beta_1 \geq 0 \; ; \; \mp \beta_6 \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_6 \geq 0$$

$$\left(\frac{1}{2}\nu \mp \beta_1 \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_1\right)\left(\mp \beta_6 \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_6\right) - \frac{1}{4}\left(\tau \mp \beta_7 \mp \frac{\theta_{(e)}}{\theta_{(s)}}\gamma_7\right)^2 \geq 0$$

$$\mp \beta_2 \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_2 = 0 \; , \; \mp \beta_3 \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_3 \geq 0 \; , \; \mp \beta_4 \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_4 \geq 0$$

$$\mp \beta_5 \mp \left(\frac{\theta_{(e)}}{\theta_{(s)}}\right)\gamma_5 \geq 0 \tag{4.11}$$

where it is understood that plus sign is for the solid continuum and the minus sign for the electrons.

The field equations from (2.2) to (2.28), with constant and equal temperatures, are given by

$$\frac{\partial \rho_{(e)}}{\partial t} + \operatorname{div}\left(\rho_{(e)}\, \underset{\sim}{V}_{(e)}\right) = 0 \tag{4.12}$$

$$- \underset{\sim}{\nabla}\Pi_{(e)} + q_{(e)}\, \underset{\sim}{E}_{(e)} + \rho_{(e)}\left(\underset{\sim}{f}_{(e)} - \overset{\prime}{\underset{\sim}{v}}_{(e)}\right) = \nu\, \underset{\sim}{u}_{(es)} - \tau\, \underset{\sim}{P}_{(s)} \tag{4.13}$$

$$\frac{\partial \rho_{(s)}}{\partial t} + \operatorname{div}\left(\rho_{(s)}\, \underset{\sim}{V}_{(s)}\right) = 0 \tag{4.14}$$

$$(\lambda+\mu)\underset{\sim}{\nabla}\underset{\sim}{\nabla}\cdot\underset{\sim}{u} + \mu\underset{\sim}{\nabla}^2\underset{\sim}{u} + q^t_{(s)}\underset{\sim}{E}_{(s)} + \left(\overset{*}{\underset{\sim}{P}}_{(s)}\times\underset{\sim}{B}\right)$$
$$+ \underset{\sim}{\nabla}\cdot\left[2\, \underset{\sim}{P}_{(s)}\otimes\underset{\sim}{E}_{(s)} + \underset{\sim}{M}_{(s)}\otimes\underset{\sim}{B} - \frac{1}{2}\underset{\sim}{P}_{(s)}\cdot\underset{\sim}{E}_{(s)}\underset{\sim}{g}\right] + \rho_{(s)}\left(\underset{\sim}{f}_{(s)} - \overset{\prime}{v}_{(s)}\right)$$
$$= \nu\, \underset{\sim}{u}_{(se)} + \tau\, \underset{\sim}{P}_{(s)} \tag{4.15}$$

where $q^t_{(s)}$ and $\pi_{(e)}$ are defined by

$$q^t_{(s)} \equiv q_{(s)} - \underset{\sim}{\nabla}\cdot P_{(s)} \quad , \quad \Pi_{(e)} = \rho^2_{(e)} \frac{\partial \Psi_{(e)}}{\partial \rho_{(e)}} \tag{4.16}$$

In addition, we have Maxwell's equations

(4.17) $$\nabla\times\underset{\sim}{E} + \frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} = 0 \quad , \quad \nabla\times\underset{\sim}{H} = \frac{1}{c}\frac{\partial \underset{\sim}{D}}{\partial t} + \frac{4\pi}{c}\underset{\sim}{J}$$

(4.18) $$\nabla\cdot\underset{\sim}{D} = 4\pi q \quad , \quad \nabla\cdot\underset{\sim}{B} = 0$$

where $\underset{\sim}{H}$ and $\underset{\sim}{D}$ are defined by

(4.19) $$\underset{\sim}{H} \equiv \underset{\sim}{B} + 4\pi\left(\frac{\underset{\sim}{v}_{(s)}}{c}\,\underset{\sim}{P}_{(s)} - \underset{\sim}{M}_{(s)}\right) \quad , \quad \underset{\sim}{D} \equiv \underset{\sim}{E} + 4\pi\,\underset{\sim}{P}_{(s)}$$

These equations together with corresponding jump conditions may be used to determine the electromagnetic and mechanical fields completely.

5. EQUATIONS OF ELASTIC CONDUCTORS

To obtain a continuum theory of elastic conductors, we assume that the inertia and thermodynamic pressure of electronic continuum are small as compared to the inertia and stress of elastic continuum. Under these assumptions, by summing equations (4.13) and (4.15) and neglecting the electron inertia and pressure, we obtain the field equations of elastic conductors

$$(5.1)\qquad (\lambda + \mu)\nabla\nabla \cdot u + \mu\nabla^2 u + q^t E + P^* \times B + \nabla\cdot[2P \otimes E + M \otimes B - \frac{1}{2} P \cdot Eq]$$
$$+ i \times B = \rho(f - \dot{v})$$

and Maxwell's equations are

$$(5.2)\qquad \begin{aligned} &\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0 \quad , \quad \nabla \times H = \frac{1}{c}\frac{\partial D}{\partial t} + \frac{4\pi}{c} J \\ &\nabla \cdot D = 4\pi q \qquad , \quad \nabla \cdot B = 0 \end{aligned}$$

where for brevity we wrote

$$(5.3)\qquad \begin{aligned} &i \equiv q_{(e)} u^{(es)}, \ E \doteq E_{(s)} \ , \ P \equiv P_{(s)}, \ M \equiv M_{(s)} \\ &f \equiv f_{(s)} \ , \ \rho \equiv \rho_{(s)} \ , \ v \equiv v_{(s)}, \ q^t = q_{(e)} + q_{(s)} - \nabla \cdot P_{(s)} \end{aligned}$$

Here, it should be noted that the conduction current i is still an unknown, but we have the equations of balance of linear momentum for electrons

$$(5.4)\qquad q_{(e)} E_{(e)} = \nu u_{(es)} - \frac{\tau}{\chi_1} \xi$$

Recalling the definition of conduction current and $(2.8)_1$, equation (5.4) may be written as

$$(5.5)\qquad (\frac{\tau}{\chi_1} + q_{(e)})\underset{\sim}{E} + \underset{\sim}{i} \times \underset{\sim}{B} = \frac{\nu}{q_{(e)}} \underset{\sim}{i}$$

The solution of $\underset{\sim}{i}$ from (5.5) is found to be

$$(5.6)\qquad i_k = \sigma_{k\ell} E_\ell$$

where $\sigma_{k\ell}$ is the conductivity tensor given by

$$(5.7)\qquad \sigma_{ij} = \frac{\sigma_\nu}{1 + \frac{B^2 q_e^2}{\nu^2}} [\delta_{ij} + \frac{q_e^2}{\nu^2} B_i B_j + \frac{q_e}{\nu} \varepsilon_{ijk} B_k]$$

with

$$(5.8)\qquad \sigma_0 = q_{(e)} (\tau \chi_2^{-1} + q_e)/\nu$$

In (5.7), the first term on the right-hand side denotes the current parallel to the effective electric field, and the second and third terms give the currents parallel to the magnetic field and perpendicular to the magnetic and electric fields. The last type of current is known as the Hall current. As might be seen from (5.7), the conductivity tensor $\sigma_{k\ell}$ is, in general, a function of the magnetic field and the density of electron gas.

Further simplification of (5.7) results, if the following condition holds, i.e.,

$$(5.9)\qquad \frac{|\underset{\sim}{B}|\ |q_{(e)}|}{\nu} \ll 1$$

This condition holds if the gyrofrequency of electrons is small as compared to the collision frequency ν. For this case, the conductivity tensor reduces to

(5.10) $$\sigma_{ij} = \sigma_0 \delta_{ij}$$

where σ_0 is called the conductivity constant.

In this case the conduction current becomes

(5.11) $$i_k = \sigma_0 E_k$$

and this is none other than the generalized Ohm's law.

6. SPHERICAL CAVITY SUBJECT TO CONSTANT ELECTROSTATIC POTENTIAL

In this section we give the solution of a problem concerned with a spherical cavity subject to a constant electrostatic potential Φ_0 and located in an infinite solid plasma. This type of problem, known as electrostatic probe, is widely used in gaseous plasma to determine certain characteristics of probes. It is hoped that the solution of this problem, together with an experiment which can be performed in a manner similar to gaseous probe, will be helpful in determining the material constants appearing in the present derivation. Since the problem is electrostatic in nature, all magnetic and collisional effects are negligible. The field equations (4.13), (4.15), and $(4.18)_1$, neglecting all the second order terms, reduce to:

(6.1) $$-\nabla \pi_{(e)} + q_{(e)} \underset{\sim}{E} + \tau \underset{\sim}{P}_{(s)} = 0$$

(6.2) $$(\lambda + \mu)\nabla\nabla \cdot \underset{\sim}{u} + \mu\nabla^2 \underset{\sim}{u} + q_{(s)} \underset{\sim}{E} - \tau \underset{\sim}{P}_{(s)} = 0$$

(6.3) $$\nabla \cdot \underset{\sim}{D} = 4\pi q$$

with

(6.4) $$\underset{\sim}{D} = \varepsilon \underset{\sim}{E} \quad , \quad q \equiv q_{(s)} + q_{(e)}$$

where ε is the dielectric constant defined by

$$\varepsilon \equiv 1 + \chi_1^{-1}$$

It is convenient to work with number densities $N_{(a)}$ defined as

$$\rho_{(e)} = m_e N_{(e)} \quad , \quad q_{(e)} = -eN_{(e)}$$

(6.5)

$$\rho_{(s)} = m_s N_{(s)} \quad , \quad q_{(s)} = ZeN_{(s)}$$

where m_e, m_s, e and Z are respectively the masses of a particle in electron and solid continua, electronic charge, and the number of valance electrons in the structure of an ionic lattice cell.

In order to proceed further, one must know the forms of $\pi_{(e)}$ and τ. For this particular problem they are assumed to be of the forms

(6.6) $$\pi_{(e)} = K\theta_{(e)}N_{(e)} \quad , \quad \tau = e(Z-\sigma)N_{(s)}$$

where K is the Boltzmann constant, $\theta_{(e)}$ is the electron temperature, and σ is a new constant which characterizes the interaction between the ionic lattice and electronic charge.

Introducing the electrostatic potential ϕ (as usual), and using (6.5) and (6.6) in (6.1) to (6.3), the field equations take the following form

(6.7) $$K\theta^{(e)} \frac{dN_{(e)}}{dr} - eN_{(e)} \frac{d\phi}{dr} + e(Z-\sigma)N_{(s)} \frac{d\phi}{dr} = 0$$

(6.8) $$(\lambda + 2\mu) \frac{d\Delta}{dr} - eZN_{(s)} \frac{d\phi}{dr} + e(Z-\sigma)N_{(s)} \frac{d\phi}{dr} = 0$$

(6.9) $$\nabla^2\phi = \frac{4\pi e}{\varepsilon} (N_{(e)} - ZN_{(s)})$$

where Δ is the volume dilatation defined by

$$\Delta \equiv \frac{1}{r^2}\frac{d}{dr}(r^2 u_r)$$

Recalling the relation

$$N_{(s)} = N^0_{(s)}(1 - \Delta)$$

The equation (6.8) may be written as

$$(6.10) \qquad \frac{(\lambda+2\mu)}{N^0_{(s)}}\frac{dN_{(s)}}{dr} + e\sigma N_{(s)}\frac{d\phi}{dr} = 0$$

where $N^0_{(s)}$ is the undeformed particle number density of the ionic lattice cell.

The equation (6.10) can be integrated to obtain

$$(6.11) \qquad N_{(s)} = A\exp[-(\frac{e\sigma N^0_{(s)}}{\lambda+2\mu})\phi]$$

Here A is an integration constant to be determined from the condition

$$(6.12) \qquad N_{(s)} \to N^0_{(s)} \quad , \quad \phi \to 0 \quad , \text{ as } r \to \infty$$

The result is

$$(6.13) \qquad N_{(s)} = N^0_{(s)}\exp[-(\frac{e\sigma N^0_{(s)}}{\lambda+2\mu})\phi]$$

Inserting (6.13) into equation (6.7), and solving the resulting nonhomogeneous differential equation for $N_{(e)}$, we obtain

$$(6.14) \qquad N_{(e)} = B\exp[\frac{e}{K\theta_{(e)}}\phi] + \frac{(Z-\sigma)(\lambda+2\mu)N^0_{(s)}}{(\lambda+2\mu)+K\theta_{(e)}\sigma N^0_{(s)}}\exp[-(\frac{e\sigma N^0_{(s)}}{\lambda+2\mu})\phi]$$

where B is a constant of integration and may be determined from the regularity condition

$$N_{(e)} \to N^{0}_{(e)} \quad , \quad \phi \to 0 \quad , \quad \text{as } r \to \infty \tag{6.15}$$

The use of (6.15) in (6.14) gives the electron number density as

$$N_{(e)} = \frac{(z-\sigma)(\lambda+2\mu)N^{0}_{(s)}}{(\lambda+2\mu)+K\theta_{(e)}N_{(s)}\sigma} \{\exp[-(-\frac{e\sigma N^{0}_{(s)}}{\lambda+2\mu})\phi - \exp[\frac{e}{K\theta_{(e)}}\phi]\}+N^{C}_{(e)}\exp[\frac{e}{K\theta_{(e)}}\phi] \tag{6.16}$$

Here $N^{0}_{(e)}$ is the initial number density of the electronic charge continuum.

If we introduce (6.13) and (6.16) into (6.9), the resultant ordinary differential equation will be highly nonlinear which cannot be treated by analytical means. This nonlinear equation can be simplified by the following arguments which are similar to the one used in the gaseous plasma, i.e.,

$$\left|\frac{e\phi}{K\theta_{(e)}}\right| << 1 \quad , \quad \left|\frac{e\sigma N^{0}_{(s)}}{\lambda+2\mu}\phi\right| << 1 \tag{6.17}$$

which states that the thermal energy of electronic charge and the elastic energy of the ionic lattice are very large as compared to the electrostatic potential of the medium. This assumption is, in particular, valid at far distances from the surface of the spherical probe. If this is the case, one can expand $N_{(e)}$ and $N_{(s)}$ into a power series of ϕ. Retaining only the first two terms of the series and introducing the result into equation (6.9), we obtain

$$\nabla^2\phi = K_D^2\phi \tag{6.18}$$

where K_D^2 is defined by

$$K_D^2 = \frac{4\pi e^2}{\varepsilon} \left[\left(\frac{1}{K\theta_{(e)}} + \frac{Z\sigma N^0_{(s)}}{\lambda+2\mu} \right) \left(1 - \frac{(Z-\sigma)(\lambda+2\mu)N^0_{(s)}}{(\lambda+2\mu)+K\theta_{(e)}\sigma N^0_{(s)}} \right) \right]$$

In obtaining the equation (6.18), we have required the charge neutrality at the initial state, i.e.,

(6.19) $$N^0_{(e)} = ZN^0_{(s)}$$

In addition, we require that $K_D^2 \geq 0$. This condition is known as the Bohm shielding criteria.

The solution of (6.18),with vanishing ϕ at infinity, is given by

(6.20) $$\phi = C\frac{e^{-K_D r}}{r}$$

Here C is a constant of integration which may be determined by equating the potential to the given potential ϕ_0 at the surface of the probe. The result is:

(6.21) $$\phi = \phi_0 \, e^{-K_D(r-a)} \left(\frac{a}{r}\right)$$

where a is the radius of the metallic probe.

From the definition of dilatation and its relation to the number density of a solid continuum, one can find the radial component of displacement satisfying the stress free boundary condition. The final result is given by

(6.22) $$u_r = \left[\frac{a^3}{4\mu} + \frac{1}{\lambda+2\mu}\left(\frac{a^2}{K_D} + \frac{a}{K_D^2}\right)\right]\frac{e\sigma\phi_0}{r^2} - \frac{e\sigma\phi_0 a}{\lambda+2\mu}\left(\frac{1}{K_D r} + \frac{1}{K_D^2 r^2}\right)e^{-K_D(r-a)}$$

$$u_\theta = u_\phi = 0$$

Similarly, the approximate number densities are:

$$N_{(s)} \doteq N^0_{(s)}\left[1 - \frac{e\sigma\Phi_0 a}{\lambda+2\mu}\,\frac{e^{-K_D(r-a)}}{r}\right]$$

(6.23)

$$N_{(e)} \doteq N^0_{(e)}\left[1 + Sa\Phi_0\,\frac{e^{-K_D(r-a)}}{r}\right]$$

where S is defined as

$$S \equiv \frac{e}{K\theta_{(e)}} - e\left(\frac{\sigma}{Z(\lambda+2\mu)} + \frac{1}{K\theta_{(e)}}\right)\left(\frac{(Z-\sigma)(\lambda+2\mu)}{(\lambda+2\mu)Z+K\theta_{(e)}N^0_{(e)}\sigma}\right)$$

From these equations one can evaluate the values of displacement and the number of densities on the surface of the sphere. These are:

$$u_r(a) = \frac{e\sigma\Phi_0 a}{4\mu}\ ,\ N_{(s)}(a) = N^0_{(s)}\ \left(1 - \frac{e\sigma\Phi_0}{\lambda+2\mu}\right)$$

(6.24)

$$N_{(e)}(a) = N^0_{(e)}(1 + Sa\Phi_0\ )$$

From physical considerations, one expects that

$$\frac{\sigma}{\lambda+2\mu} \geq 0 \quad , \quad S \geq 0$$

This requirement is consistent with the one posed for K_D^2.

If the same problem was solved by use of the classical theory of elastic conductors, the result would be

(6.25) $$\Phi = \Phi_0\left(\frac{a}{r}\right)$$

which is equivalent to equation (6.21) as K_D^2 approaches to zero.

As a concluding remark, it is worthwhile to cite that the present approach not only gives the distribution of electrostatic potential but also the variations of the densities of the electronic and ionic species that form an elastic conductor.

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